Marking Scheme Q2 (10 points)

Part A (5.6 pt)

A-1		1.5 pt
(a)	$\vec{E}(x,y,z) = \frac{-\lambda x}{4\epsilon_0 R^2}\hat{x} + \frac{-\lambda y}{4\epsilon_0 R^2}\hat{y} + \frac{\lambda z}{2\epsilon_0 R^2}\hat{z}$ (1.0 pt) [z-component (0.5 pt), x- and y- components (0.5 pt), wrong coefficient for each component (-0.1 pt), wrong sign for each component (-0.2 pt)]	
(b)	$\omega_x = \omega_y = \sqrt{\frac{Q\lambda}{4\epsilon_0 R^2 m}}$ (0.5 pt)	
A-2	$a = \frac{Qu}{2\epsilon_0 R^2 m\Omega^2}$ (0.2 pt) $k = \sqrt{\frac{Q\lambda_0}{2\epsilon_0 R^2 m}}$ (0.2 pt)	0.4 pt
A-3	$\ddot{q} = pa\Omega^2 \cos \Omega t$ (1.0 pt) [each of the 3 approximations (0.3 pt), the final equation (0.1pt)] $q = -pa \cos \Omega t$ (0.8 pt) [general solution (0.4 pt), fixing the free parameters in the general solution each (0.2 pt)]	1.8 pt
A-4	$\ddot{p}(t) = \left(k^2 - \frac{a^2\Omega^2}{2}\right)p$ (1.2 pt) [Correct approach (0.6 pt), Correct result (0.6 pt)] $\Omega > \sqrt{2}\frac{k}{a}$ (0.3 pt)	1.5 pt
A-5	$k = 2 \times 10^5$ rad/s (0.2 pt) $\Omega_{\min} \simeq 7 \times 10^6$ rad/s (0.2 pt) [inappropriate number of significant figures (-0.1 pt)]	0.4 pt

Part B (4.4 pt)

B-1	$\Gamma = \frac{1}{\tau}$ (0.5 pt) [Answers with different numerical coefficients should be considered as correct answers]	0.5 pt

B-2	$s_+ = s_\mathrm{L} + \alpha\omega_\mathrm{L}\frac{v}{c}$ (0.5 pt) $s_- = s_\mathrm{L} - \alpha\omega_\mathrm{L}\frac{v}{c}$ (0.5 pt) [correct Doppler shift each (0.3 pt), final answer each (0.2 pt)] $\pi_+ = s_+ \times (-\hbar k_+)$ (0.1 pt) $\pi_- = s_- \times (+\hbar k_-)$ (0.1 pt) $F = -(2\alpha\hbar k_\mathrm{L}^2)v$ (0.5 pt)	1.7 pt
B-3	$\begin{cases} p = 0 \\ p = +2\hbar k_\mathrm{L} \end{cases}$ (0.5 pt) [one correct answer (0.3 pt)] $P_\mathrm{in} = \frac{\hbar^2 k_\mathrm{L}^2}{m\tau}$ (0.5 pt)	1.0 pt
B-4	$P_\mathrm{out} = -2\alpha\hbar k_\mathrm{L}^2 v^2$ (0.3 pt) $\overline{v^2} = \frac{\hbar\Gamma}{2\alpha m}$ (0.3 pt) $T = \frac{\hbar\Gamma}{2\alpha k_\mathrm{B}}$ (0.2 pt) [Answers with different numerical coefficients should be considered as correct answers]	0.8 pt
B-5	$T = 2 \times 10^{-4}\ \mathrm{K}$ (0.4 pt) [according to the coefficient used in the part B.4, the resulting temperature might be different.]	0.4 pt